Lesson Four Purpose

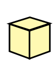
- Understand concrete and symbolic representations of real numbers in real-world situations. (MA.A.1.4.3)
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results. (MA.A.4.4.1)
- Use concrete and graphic models to derive formulas for finding perimeter, area, surface area, circumference, and volume of two- and three-dimensional shapes, including rectangular solids, cylinders, cones, and pyramids. (MA.B.1.4.1)
- Solve real-world and mathematical problems, involving estimates of measurements, including length, perimeter, and area and estimate the effects of measurement errors on calculations. (MA.B.3.4.1)

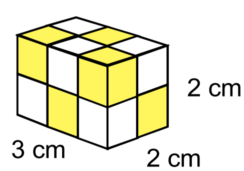
Volume

Volume of a Solid

Solid figures are **three-dimensional** figures that have length, width, and height and completely enclose a portion of space. *Solid figures* with flat surfaces are called **polyhedrons**. *Polyhedrons* are *three-dimensional* figures in which all surfaces are polygons. We will study two types of polyhedrons: **prisms** and **pyramids**.

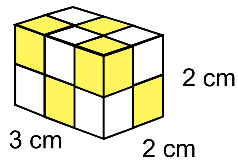
The **volume (*V*)** of a solid is the amount of space a solid contains. *Volume* is measured in **cubic units**. Therefore, the number of *cubic units* it takes to fill the solid is its volume.

Look at the solid figure with three **cubes**  across the front and two cubes on the end. We can think of the bottom layer as being made up of six cubes. There are two layers of cubes, so the *volume* (or number of cubes needed to fill the box) will be 12 cubic centimeters, abbreviated 12 cm^3.



A **rectangular prism** has six **faces**. A *face* is a flat surface of a solid figure. Each face on a *rectangular prism* is a rectangle.

This rectangular prism has a length of 3, a width of 2, and a height of 2.

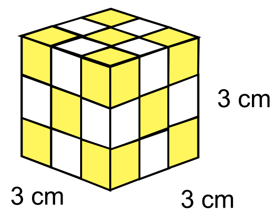


Example: *rectangular prism*

Volume = length x width x height

$V = lwh$
$V = (3)(2)(2)$
$V = 12 \text{ cm}^3$

If the solid has the same value for its length, width, and height, we have a cube.



Volume = $(3)(3)(3) = 27 \text{ cm}^3$

Volume of a Cylinder

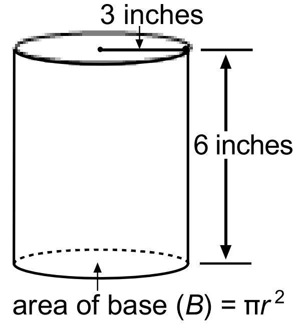
A **cylinder** is a three-dimensional figure with two parallel congruent circular bases.

To find the volume of a *cylinder*, we multiply the *area of the base* (*B*) by the height (*h*). Note that the base of a cylinder is a circle.

Example: *cylinder*

Volume = *B* x *h* (*B* is area of base)(*h* is height of cylinder)

$V = \pi r^2 h$
$V \approx (3.14)(3^2)(6)$
$V \approx (3.14)(9)(6)$
$V \approx 169.56$ cubic inches *or*
169.6 inches^3 *or* abbreviated as 169.6 in.^{3}
(rounded to the nearest tenth)



Volume of a Cone

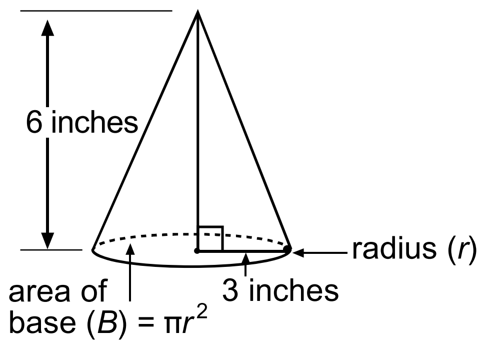
A **cone** is a three-dimensional figure with one circular base in which a curved surface connects the base to the vertex. A *right circular cone* has a center that forms a line perpendicular to its circular base. Consider a right circular cone with the same radius (r) and height (h) as the cylinder described on the previous page.

The cone's volume is $\frac{1}{3}$ of the volume of the above cylinder with a radius of 3 inches and a height of 6 inches. Therefore, to figure the cone's volume, we take $\frac{1}{3}$ of the volume of the cylinder.

Example: *cone*

Volume = $\frac{1}{3}$(volume of cylinder)

$$V = \frac{1}{3}\pi r^2 h$$
$$V = (\tfrac{1}{3})(\pi)(3^2)(6)$$
$$V = (\tfrac{1}{3})(\pi)(9)(6)$$
$$V = (\tfrac{1}{3})(54)(\pi)$$
$$V = 18\pi \text{ inches}^3$$



Frequently, answers are shown with the Greek letter (π) pi rather than calculating the decimal answer. This is useful when comparing areas and volumes of circular shapes. A decimal representation of our answer is

$$V \approx 18(3.14) \text{ inches}^3$$
$$V \approx 56.52 \text{ inches}^3 \textit{ or } 56.52 \text{ in.}^3$$

Volume of a Prism and Square Pyramid

A *prism* is a three-dimensional figure with congruent, polygonal bases and lateral faces that are all parallelograms. A *rectangular prism* is a six-sided prism whose faces are all rectangular.

A *pyramid* is a three-dimensional figure with a single polygonal base and triangular faces that meet at a common point (vertex). A **square pyramid** is a pyramid with a square base and four faces that are triangular.

A square pyramid's volume is $\frac{1}{3}$ of the volume of a prism with a square base and a height equal to a prism's height. See the drawings below. Note that the bases are squares with areas of 36 square inches. Heights of the prism and pyramid are 10 inches.

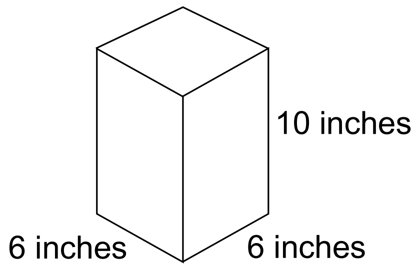
Example: *rectangular prism*

Volume = length x width x height

$V = (6)(6)(10)$

$V = (36)(10)$

$V = 360$ inches^3 (in.^{3})



Example: *square pyramid*

Volume = $\frac{1}{3}$(area of base) x h

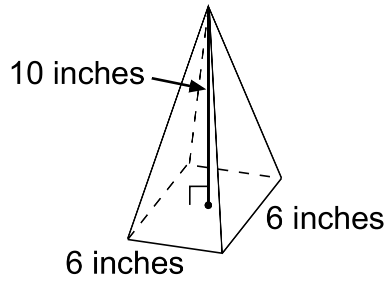
$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(6 \times 6)(10)$

$V = \frac{1}{3}(36)(10)$

$V = 12(10)$

$V = 120$ inches^3 (in.^{3})



Volume of a Sphere

A **sphere** is a three-dimensional figure in which all points on the surface are the same distance from the center.

The volume of a *sphere* is calculated using the following formula.

Example: *sphere*

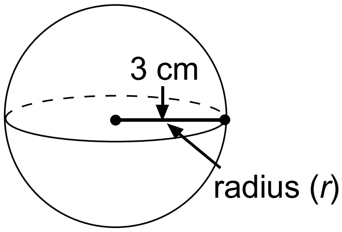
Volume $= \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(3^3)$

$V = \frac{4}{3}(27)\pi$

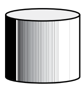
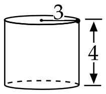
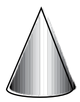
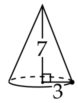
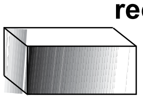
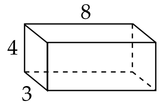
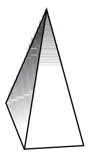
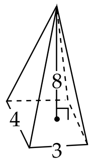
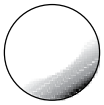
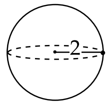
$V = 36\pi \text{ cm}^3$

($or \approx 113.04 \text{ cm}^3$)



Let's Summarize:

Volume (V) is the amount of space occupied in a three-dimensional figure. Volume is expressed in cubic units.

Mathematical Formulas for Volume (V)		
figure	**formula**	**example**
 **right circular cylinder**	$V = \pi r^2 h$	 $V \approx (3.14)(3^2)(4)$ $V \approx 113.04$ cubic units
 **right circular cone**	$V = \frac{1}{3}\pi r^2 h$ *or* $V = \frac{1}{3}Bh$	 $V \approx \frac{1}{3}(3.14)(3)^2(7)$ $V \approx 66$ cubic units
 **rectangular solid**	$V = lwh$	 $V = 8(3)(4)$ $V = 96$ cubic units
 **square pyramid**	$V = \frac{1}{3}lwh$ *or* $V = \frac{1}{3}Bh$	 $V = \frac{1}{3}(3)(4)(8)$ $V = \frac{96}{3} = 32$ cubic units
 **sphere**	$V = \frac{4}{3}\pi r^3$	 $V \approx \frac{4}{3}(\frac{22}{7})(2^3)$ $V \approx 33\frac{11}{21}$ cubic units

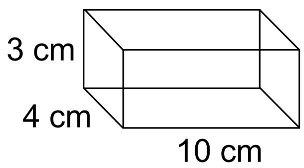
Key				
B = area of base	h = height	l = length	r = radius	V = volume
π = pi Use 3.14 or $\frac{22}{7}$ for π.		πr^2 = area of circular base		

Note: Appendix C contains a list of various shapes with formulas for finding area, volume, and surface area. You will *not* need to memorize all of these formulas.

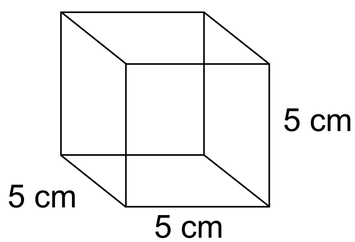
Practice

Find the **volume** *of each* **prism**. *Use the formula* $\boldsymbol{V = lwh}$.

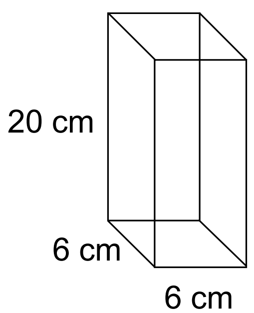
1.



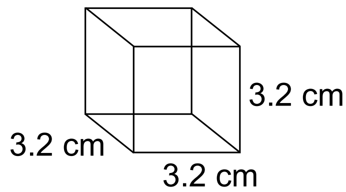
2.



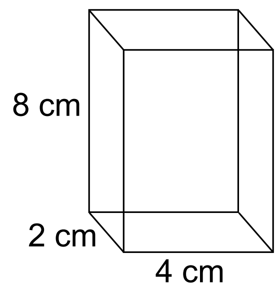
3.



4.



5.



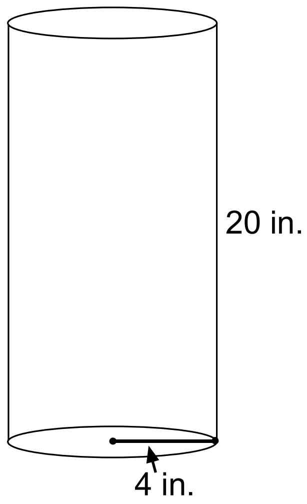
Find the **volume** *of each* **prism**. *Use the formula* $V = lwh$.

	length	**width**	**height**
6.	3 m	2 m	30 m
7.	10 ft	8 ft	15 ft
8.	16.2 cm	10 cm	1.5 cm
9.	$3\frac{1}{2}$ in.	2 in.	8 in.
10.	$12\frac{1}{2}$ m	8 m	3 m

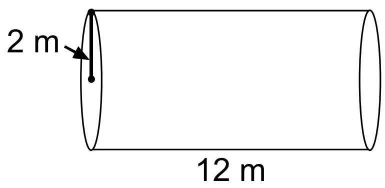
Practice

Find the **volume** *of each* **cylinder**. *Use the formula* $\mathbf{V = \pi r^2 h}$. **Round answers to the nearest tenth.**

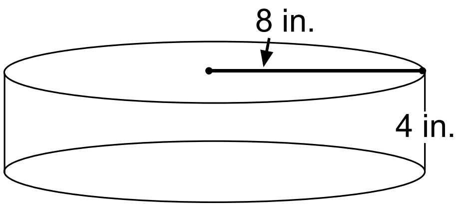
1.



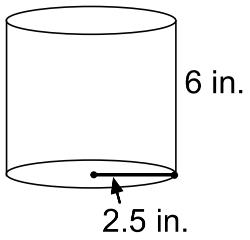
4.



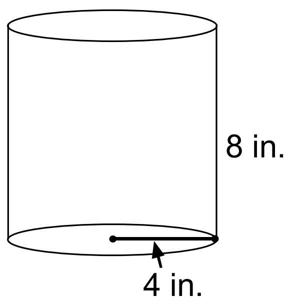
2.



5.



3.



Find the **volume** *of each* **cylinder**. *Use the formula* $\boldsymbol{V} = \pi r^2 \boldsymbol{h}$. **Round answers to the nearest tenth.**

6. radius, 2 in.; height, 10 in.; use 3.14 for π

7. diameter, 20 m; height 6 m; use 3.14 for π

   Remember: The radius is $\frac{1}{2}$ of the diameter.

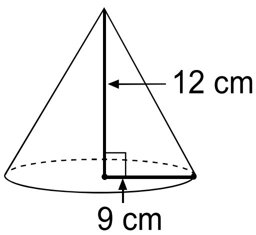
8. diameter, 5 ft; height 8 ft; use 3.14 for π

9. radius, 7 m; height 12 m; use $\frac{22}{7}$ for π

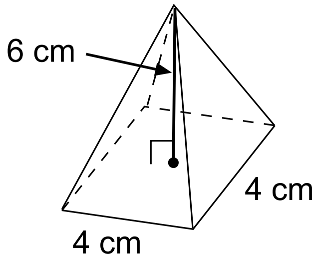
10. Refer to problems 2 and 3. Do the two cylinders have the same volume? ____________ If not, which volume is greater and by how much? ____

Find the **volume**. *Use the formula* $\frac{1}{3}\pi r^2 h$ *and* **round answer to the nearest tenth**.

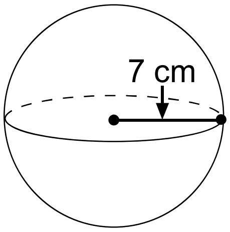
11.



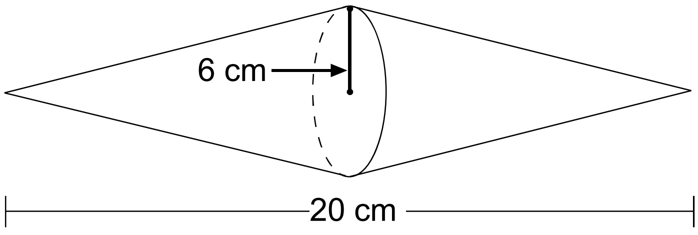
Find the **volume**. *Use the formula* $\frac{1}{3}Bh$ *and* **round answer to the nearest tenth**.

12. 

Find the **volume**. *Use the formula* $\frac{4}{3}\pi r^3$ *and* **round answer to the nearest tenth**.

13. 

14. Two identical right circular cones have been placed with their bases touching to create the sculpture shown in the drawing below. The radius of each base is 6 cm and the total length of the object is 20 cm.

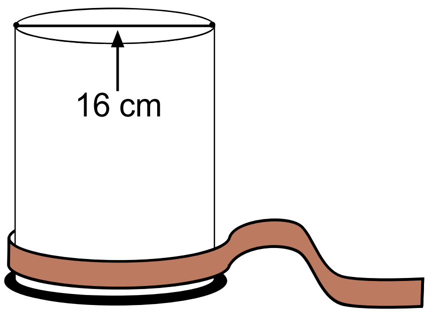


What is the volume, in cubic centimeters, of the sculpture?

Answer the following.

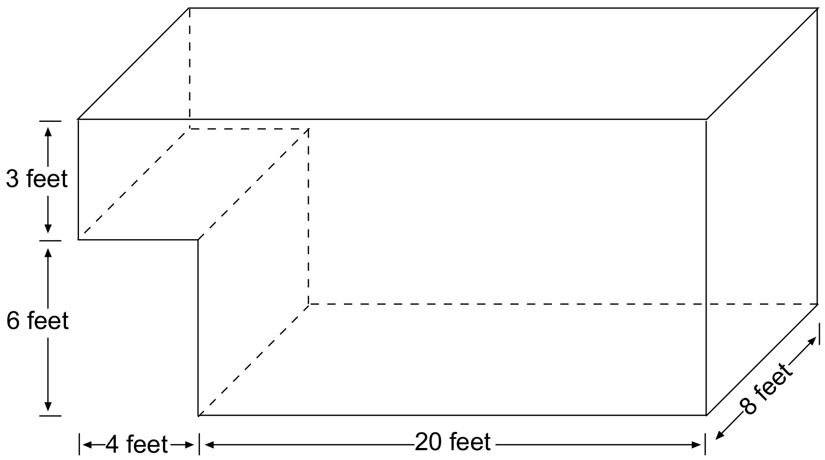
15. A cylindrical column 16 centimeters in diameter is strengthened by wrapping one steel band around the base of the column, with no overlap. What should be the length in centimeters of the steel band?

Remember: $C = d\pi$



Challenge Problem:

16. The diagram below shows the dimensions of the cargo area of a moving van. What is the maximum volume of cargo, in cubic feet, that can fit in the van?



Practice

Use the list below to complete the following statements.

circle	**cylinder**	**rectangular prism**
cone	**face**	**square pyramid**
cube	**prism**	**volume**
cubic units		

1. The ______________________ of a solid is the amount of space a solid contains.

2. Volume is measured in ______________________ .

3. To find the volume of a ______________________ , we multiply the area of the base by the height.

4. Each base of a *cylinder* is a ______________________ .

5. A ______________________ is a rectangular prism that has six square faces.

6. A ______________________ has six faces.

7. A ______________________ is a flat surface of a solid figure.

8. A ______________________ is a three-dimensional figure with one circular base in which a curved surface connects the base to the vertex.

9. A ________________________ is a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms.

10. A ________________________ is a pyramid with a square base and four faces that are triangular.

Match each definition with the correct term. Write the letter on the line provided.

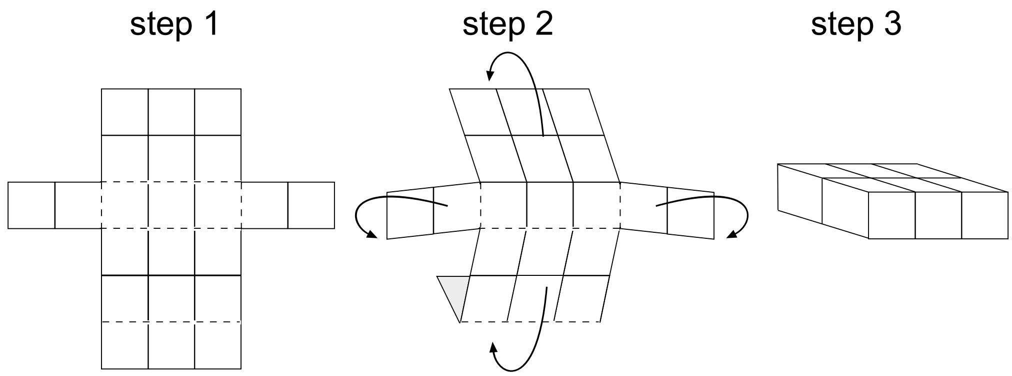
______ 11. three-dimensional figures that completely enclose a portion of space		A. polyhedron
______ 12. a three-dimensional figure in which all points on the surface are the same distance from the center		B. pyramid
______ 13. a three-dimensional figure in which all surfaces are polygons		C. solid figures
______ 14. existing in three dimensions; having length, width, and height		D. sphere
______ 15. a three-dimensional figure (polyhedron) with a single base that is a polygon and whose faces are triangles and meet at a common point (vertex)		E. three-dimensional (3-dimensional)

Surface Area of Three-Dimensional Shapes

Using graph paper, we can see that two-dimensional shapes can be cut out and folded to create three-dimensional shapes. We call this two-dimensional pattern a **net**. A *net* is a plan which can be used to make a model of a solid.

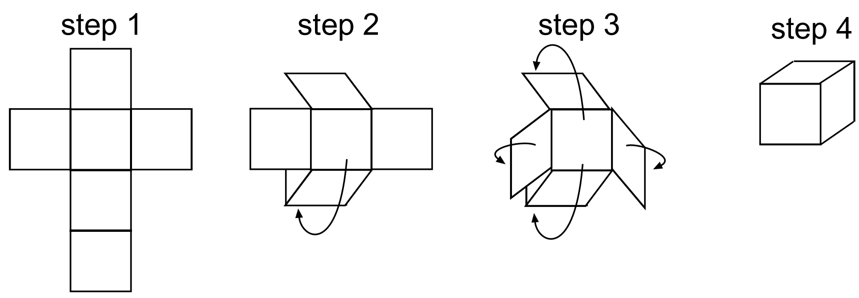
Example 1: *rectangular prism*

Draw this figure, cut it out, fold on the dashed lines, and tape it together to form a rectangular prism.



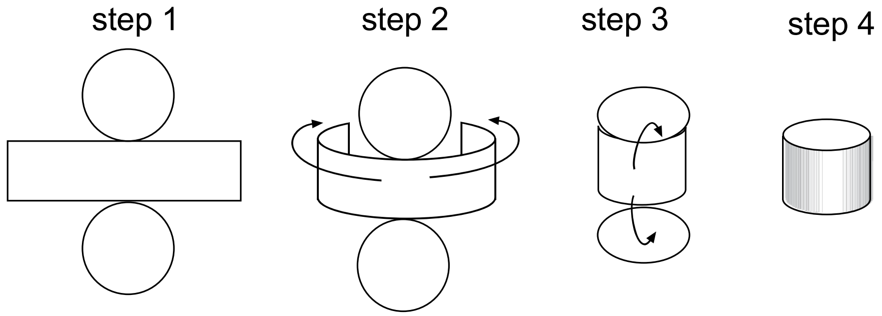
Example 2: *cube*

Draw this figure, cut it out, fold, and tape it together to make a cube.



Example 3: *cylinder*

Draw this figure, cut it out, wrap the rectangle around the circles, and tape it together to form a cylinder.



Surface Area and Lateral Area

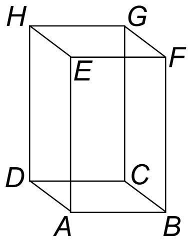
Surface area (*S.A.*) includes the areas of the faces of a figure that create a geometric solid. *Surface area* is expressed in *square units*.

The surface area of a rectangular prism is the sum of the areas of the bases and faces of the prism. Adding all the areas of the faces gives you the area of the entire surface of the box—the surface area.

Inspect the prism drawn below and we will identify its faces. There are the **lateral** faces (or *sides* of the shape), which are rectangles. *Lateral* refers only to the surfaces on the figure, not the *base*. Note the four rectangular faces. We can find the area of each face and add each of the four areas to get the *lateral area* (*L.A.*). Or we can use the perimeter of the bottom ($AB + BC + CD + DA$) times the height to calculate lateral area.

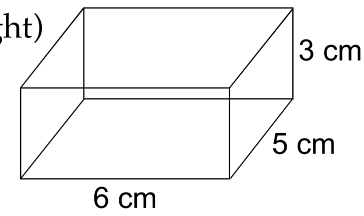
If the prism were wrapped in paper and we remove the paper, the shape would be a rectangle with length of $AB + BC + CD + DA$ and height AE (or BF or CG or DH). For total surface area, we find area of top and area of bottom and add these areas to our lateral area.

Our formula is
total surface area = 2(area of base) + perimeter of base(height).



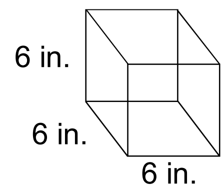
Total Surface Area of a Rectangular Prism and a Cube

Example: *rectangular prism*



$S.A. = 2(\text{area of base}) + \text{perimeter of a base(height)}$
$= 2(5 \times 6) + (22)(3)$
$= 60 + 66$
$= 126 \text{ cm}^2$

Example: *cube*



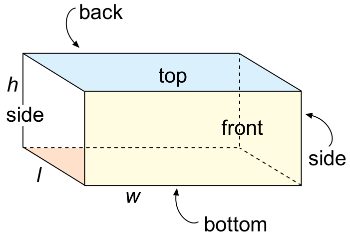
$S.A. = 2(\text{area of base}) + \text{perimeter of base(height)}$
$= 2(6 \times 6) + 24(6)$
$= 72 + 144$
$= 216 \text{ in.}^2$

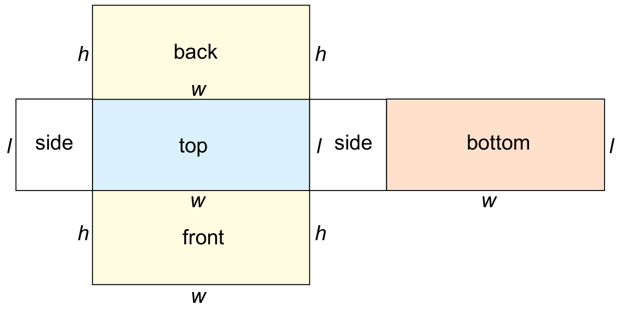
Another Way to Determine Surface Area of a Rectangular Prism and a Cube

Here's another way to determine the surface area of a prism. See the rectangular prism drawn below. Consider that a rectangular prism has six faces. You must find the area of each of its six faces:

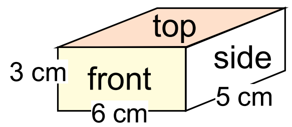
- a top and a bottom—$2(lw)$
- a front and a back—$2(hw)$
- two sides—$2(lh)$

$S.A. = 2lw + 2hw + 2lh$ or $2(lw + hw + lh)$

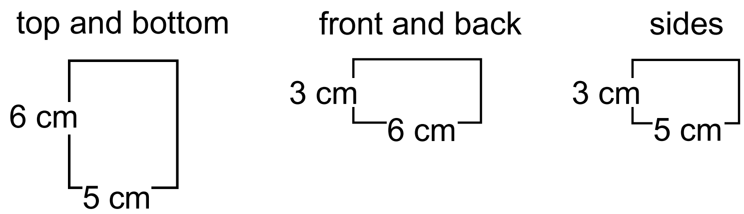




Example: *rectangular prism*



So, to find the surface area of the rectangular prism, you can make a sketch of the rectangular faces and label the dimensions.

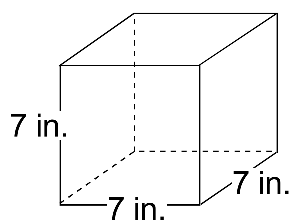


S.A. = top and bottom	+ front and back	+ sides
S.A. = $2lw$	+ $2hw$	+ $2lh$
S.A. = 2(5 • 6)	+ 2(6 • 3)	+ 2(5 • 3)
S.A. = 60	+ 36	+ 30
S.A. = 126 cm^2		

Example: *cube*

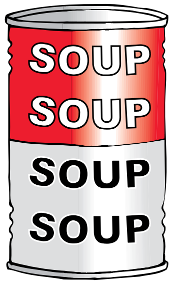
In a cube, all six faces have the same area. You only need to find the area of one face, then multiply by 6.

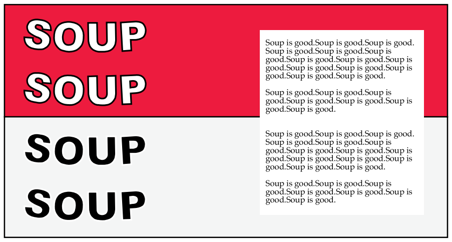
S.A. = 6(side x side)
= $6s^2$
= 6(7 • 7)
= 6(49)
= 294 $in.^2$



Total Surface Area of a Cylinder and a Sphere

To see an example of a cylinder, look at a can of soup. If we remove its wrapper by neatly cutting from top to bottom, the wrapper becomes a rectangle when we flatten it.



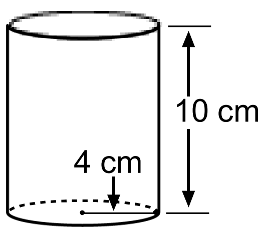


Instead of several rectangular shapes for the sides, our lateral area is a curved surface attached to a circular top and a circular bottom (the bases). Our total surface area formula is very similar to that of the prism except that our bases are circular.

Example: *cylinder*

The surface area of a cylinder is two times the circumference of the base (πr) times the height (h) plus two times the area of the base (πr^2).

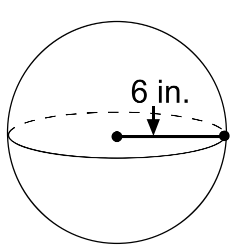
$$
\begin{aligned}
S.A. &= (2\pi r)(h) + 2(\pi r^2) \\
&= 2\pi(4)(10) + 2\pi(16) \\
&= 80\pi + 32\pi \\
&\approx 251.2 + 100.48 \\
&\approx 351.68 \text{ cm}^2
\end{aligned}
$$



Example: *sphere*

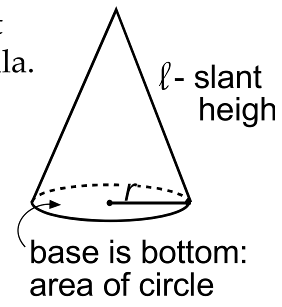
The surface area of a sphere is the circumference ($2\pi r$) times the diameter ($2r$) or ($2\pi r$)($2r$), which equals $4\pi r^2$.

$$
\begin{aligned}
S.A. &= 4\pi r^2 \\
&= 4\pi(36) \\
&= 144\pi \text{ in.}^2 \\
&\approx 452.16 \text{ in.}^2
\end{aligned}
$$



Total Surface Area of a Cone and a Square Pyramid

To find surface area for cones and pyramids, notice that only one base and the outer edge are used in our formula. The outer edge is called the **slant height** and is designated by the letter ℓ.



Example: *cone*

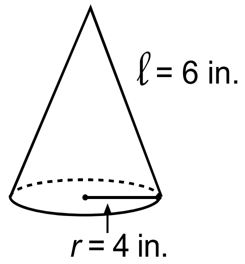
The lateral area of a cone is half of the circumference of the base ($2\pi r$) multiplied by the slant height (ℓ).

$$\text{lateral area} = \tfrac{1}{2}(2\pi r)(\ell)$$

To determine the surface area of a cone, you must find

- the lateral area [$\tfrac{1}{2}(2\pi r)(\ell)$]
- the area of the base (πr^2).

The total surface area of a cone is the lateral area of the cone [$\tfrac{1}{2}(2\pi r)(\ell)$] plus the area of the base (πr^2).



$$
\begin{aligned}
S.A. &= \tfrac{1}{2}(2\pi r)(\ell) + \pi r^2 \\
&= \pi r\ell \qquad + \pi r^2 \\
&= \pi(4)(6) \quad + \pi 4^2 \\
&= 24\pi \qquad + 16\pi \\
&= 40\pi \text{ in.}^2 \\
&\approx 125.6 \text{ in.}^2
\end{aligned}
$$

Example: *square pyramid*

The faces of a pyramid are triangles, so we use the *altitude* (or height) of a triangular face called the *slant height* (ℓ) in our formula. The lateral area of a pyramid is the sum of the areas of its lateral faces (not including the base). The slant height (ℓ) is the length of the altitude of a lateral face of a regular pyramid. To note the length of the edge of the square base, we use the symbol l.

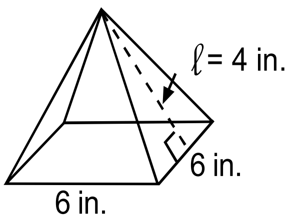
The lateral area of each triangular face is half of the length of the edge of the base (l) times the slant height (ℓ).

To determine the surface area of a square pyramid, you must find

- the lateral area of each triangular face ($\frac{1}{2}l\ell$)
- the area of the square base (l^2).

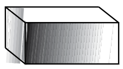
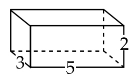
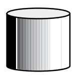
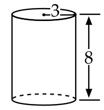
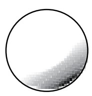
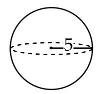
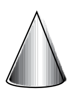
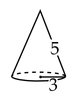
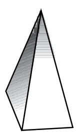
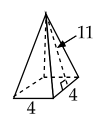
The surface area of a square pyramid is the area of four triangular faces [$4(\frac{1}{2}l\ell)$] plus the area of the base (ℓ^2).

$$\begin{aligned} S.A. &= 4(\tfrac{1}{2}l\ell) &+ l^2 \\ &= 2l\ell &+ l^2 \\ &= 2(6 \bullet 4) &+ 6^2 \\ &= 2(24) &+ 6^2 \\ &= 48 &+ 36 \\ &= 84 \text{ in.}^2 \end{aligned}$$



Let's Summarize:

Surface area (*S.A.*) is the sum of all the areas of the faces (including the bases) of a solid figure. Surface area is expressed in square units.

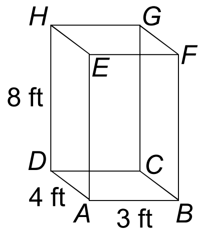
Mathematical Formulas for Surface Area (*S.A.*)		
figure	**formula**	**example**
 **rectangular solid**	$S.A. = 2(lw) + 2(hw) + 2(lh)$	 $S.A. = 2(5 \cdot 3) + 2(3 \cdot 2) + 2(5 \cdot 2)$ $S.A. = 2(15) + 2(6) + 2(10)$ $S.A. = 30 + 12 + 20$ $S.A. = 62$ square units
 **right circular cylinder**	$S.A. = 2\pi rh + 2\pi r^2$	 $S.A. \approx 2(3.14)(3 \cdot 8) + 2(3.14)(3)^2$ $S.A. \approx 150.72 + 56.52$ $S.A. \approx 207.24$ square units
 **sphere**	$S.A. = 4\pi r^2$	 $S.A. \approx 4(3.14)(5)^2$ $S.A. \approx (12.56)(25)$ $S.A. \approx 314$ square units
 **right circular cone**	$S.A. = \frac{1}{2}(2\pi r)\ell + \pi r^2$ $= \pi r\ell + \pi r^2$	 $S.A. \approx (3.14)(3)(5) + 3.14(3)^2$ $S.A. \approx 47.1 + 28.26$ $S.A. \approx 75.36$ square units
 **square pyramid**	$S.A. = 4(\frac{1}{2}l\ell) + l^2$ $= 2l\ell + l^2$	 $S.A. = 2(4 \cdot 11) + 4^2$ $S.A. = 2(44) + 4^2$ $S.A. = 88 + 16$ $S.A. = 104$ square units

Key				
h = height	l = length	r = radius	ℓ = slant height	*S.A.* = surface area
π = pi Use 3.14 or $\frac{22}{7}$ for π.				

Note: Appendix C has a list of formulas for surface area.

Practice

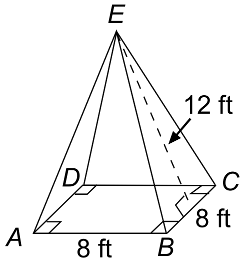
Use the examples **above** *each section to answer the items in that section. Refer to* **formulas** *in unit or in* **Appendix C** *as needed.*



Section 1

1. What kind of polygons are the lateral faces? ____________________
2. What kind of polygons are the bases? __________________________
3. What is the perimeter of the base? _____________________________
4. What is the height of the prism? _______________________________
5. If you were to paint the total surface of this prism, how many square

   feet (ft^2) need to be covered? __________________________________



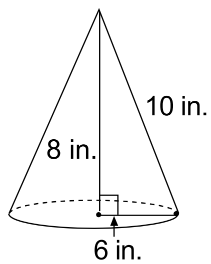
Section 2

6. What kind of polygon is the pyramid's base?__________________
7. Find the area of the base. ___________________________________
8. Name the lateral faces of the pyramid. _______________________

9. The area of the lateral surface is 4 times $\frac{1}{2}$(length of the base) *or*

2(length of the base) x ________________________________ .

10. Find the total surface area. ______________________________



Section 3

11. What is the slant height of the cone? ______________________

12. Find the area of the base. Leave π in your answer.

____

13. Find the perimeter of the base. Again, leave π in your answer.

____

14. Total surface area = lateral area + area of the base

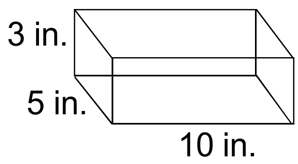
$$\begin{aligned} &= \tfrac{1}{2}(2\pi r)(\ell) && + \pi r^2 \\ &= \pi r \ell && + \pi r^2 \\ &= \pi(6)(10) && + \pi(6)^2 \\ &= 60\pi && + 36\pi \\ &= 96\pi \text{ square inches or } 96\pi \text{ in.}^2 \end{aligned}$$

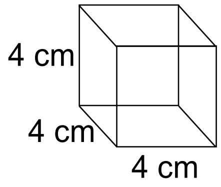
≈ ____________ in.²

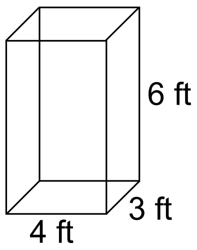
Round to the nearest tenth.

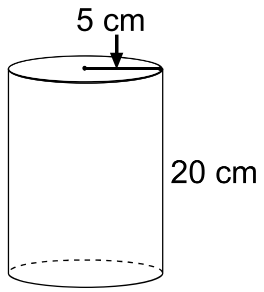
Practice

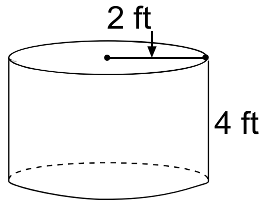
Find the total **surface area** *of the following* **solids**. *For problems using* π, **round answers to the nearest tenth**. *Refer to formulas in unit or in* **Appendix C** *as needed.*

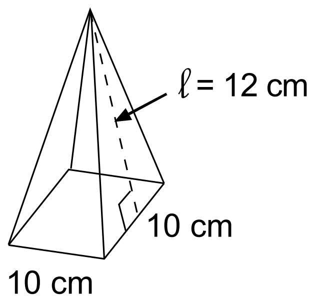
1. 

2. 

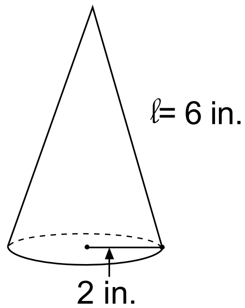
3. 

4. 

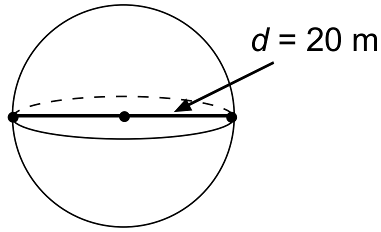
5. 

6. 

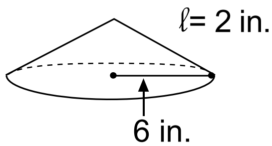
7.



9.



8.



Match each definition with the correct term. Write the letter on the line provided.

______ 10. the shortest distance from the vertex of a right circular cone to the edge of its base; the perpendicular distance from the vertex of a pyramid to one edge of its base

______ 11. a surface on the side of a geometric figure, as opposed to the base

______ 12. the sum of the areas of the faces of the figure that create the geometric solid

______ 13. a plan which can be used to make a model of a solid; a two-dimensional shape that can be folded into a three-dimensional figure

A. lateral

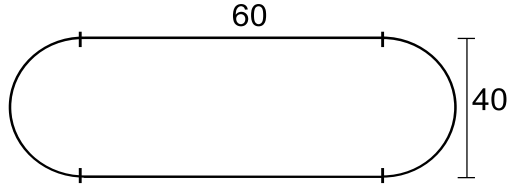
B. net

C. slant height

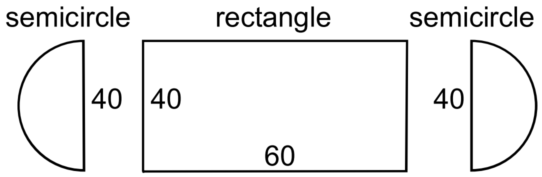
D. surface area (*S.A.*)

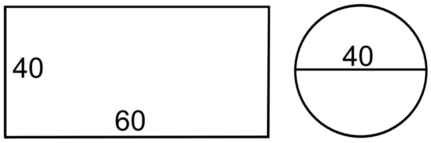
Problem Solving

How would you find the area of this odd shape?



If you look closely you can find a rectangle and 2 semicircles. If you put the semicircles together, you would, of course, have a full circle with a diameter of 40 feet. The rectangle has length 60 feet and width 40 feet.





To find the area of this odd shape you need to add the area of the circle and the area of the rectangle.

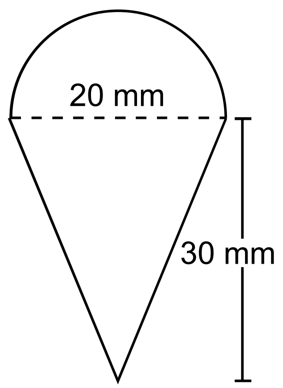
Remember: The radius is $\frac{1}{2}$ the diameter.

Area of Circle	+	**Area of Rectangle**	=	**Area of Total Shape**
$A = \pi r^2$		$A = lw$		
$(3.14)(20)^2$		$(60)(40)$		
1,256 ft^2	+	2,400 ft^2	$\approx$	3,656 ft^2

Practice

Answer the following. Refer to **formulas** *in unit or in* **Appendix C** *as needed.*

1. Find the area of this shape.

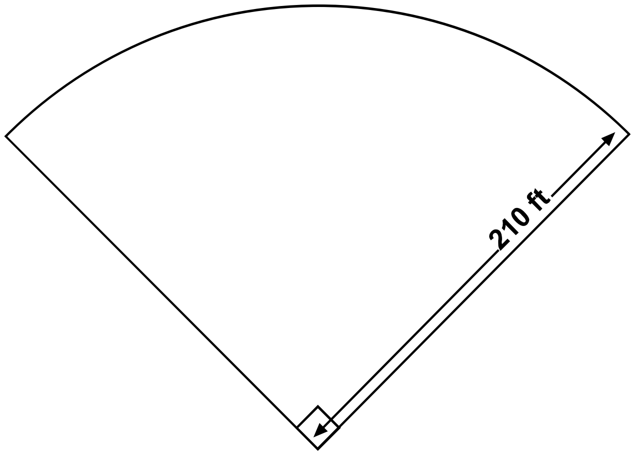


Hint: Do you see a semicircle and a triangle? Add their areas together.

Area of Semicircle	**+**	**Area of Triangle**	**=**	**Area of Total Shape**
$A = \frac{1}{2}(\pi r^2)$	+	$A = \frac{1}{2}bh$	$\approx$	

Hint: Your answer should be between 400 and 500 square millimeters (mm^2).

2. A softball field is constructed in the shape of $\frac{1}{4}$ of a circle, as shown in the diagram below:



If the coach has his players run the circular part of the field, what would be the total distance that a player would run?

Hint: Use the formula for circumference of a circle.
$C = \pi d$ *or*
$C = 2\pi r$

Remember that a player will be running only $\frac{1}{4}$ of the circle.

How many feet would a player run if he runs the circular part of the field 4 times? ______________________________

Hint: Your answer should be between 1,000 and 1,500 feet.

There are 5,280 feet in a mile. Approximately how many times would you need to run just the circular part of the field to run a mile?

____

3. The coach in problem number 2 decides to have his players run around the entire field.

Hint: circular part + 210 + 210

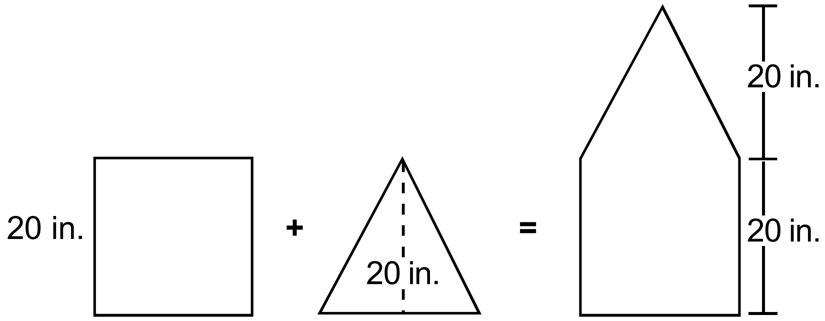
What would be the total distance that a player would run? _____

If the players run around the field 5 times:

How many feet would this be? ______________________________

Is this more than a mile or less than a mile? ___________________

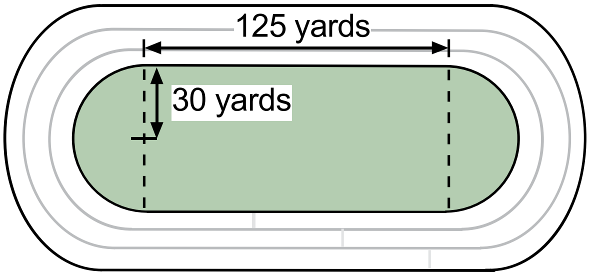
4. A square and a triangle can be combined to make an irregular pentagon.



What would be the area of this figure? _______________________

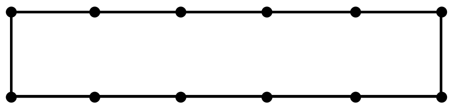
5. Susie is helping to replant the grass area inside the track at her high school. She needs to find the area of the shaded region shown in the diagram below so she can order the sod. The radius of each half-circle end of the inside of the track is 30 yards.

   What is the area, in square yards (yd^2), of the shaded region? ______



6. Ben has ordered 12 sections of fencing connected by hinges. Each section is 1 foot. Ben plans to make a pen for his dog, and he wants to give his dog the largest pen possible. Here is one possibility.

Find four other ways to build the pen. Fill in the chart below.



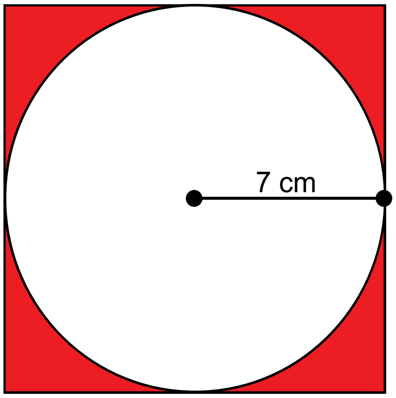
Ways to Build a Dog Pen

length	width	area (square feet—ft^2)
5	1	5

Which arrangement gives the maximum or largest area? ________

What kind of rectangle is this? ________________________

7. A circle with radius 7 cm is inscribed in the square below. Find the area of the shaded region.



Hint: To solve this problem, you have to use your imagination. If you cut the circle out with a pair of scissors, you would have the shaded part left!

area of square – area of circle = area of shaded area

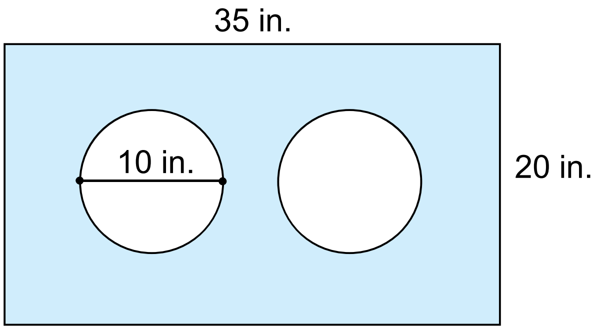
Find the answer. Be sure that you figure the length and width of the square correctly! The final answer should be between 40 and 50 square centimeters (cm^2).

____

8. Bob drills 2 holes with diameters of 10 inches from a board 35 inches by 20 inches.

Hint: rectangle – 2 circles = what is left

How much of the board is left? _______________________



Practice

Circle the letter of the correct answer. Refer to **formulas** *in unit or in* **Appendix C** *as needed.*

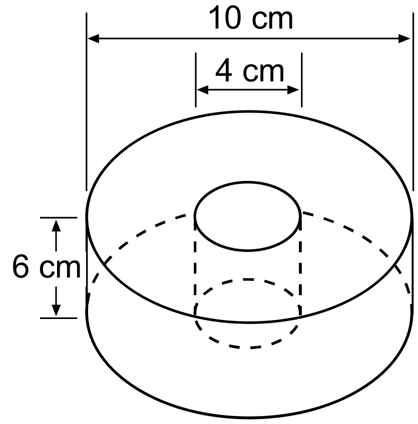
1. An engineer is designing a metal gasket for a machine. The gasket is in the shape of a cylinder with a cylindrical hole through its center. The diameter of the gasket is 10 centimeters, and its height is 6 centimeters. The diameter of the hole is 4 centimeters.

   Hint: volume of big cylinder – volume of little cylinder = volume of the gasket

   What is the volume of the gasket? ____________________________

   Your answer will be in cubic centimeters. The possible answers have π in them, so do not replace π with 3.14 in this problem. Make sure that you use the correct formula.

   a. 126π cubic centimeters
   b. 504π cubic centimeters
   c. 81π cubic centimeters
   d. 174π cubic centimeters

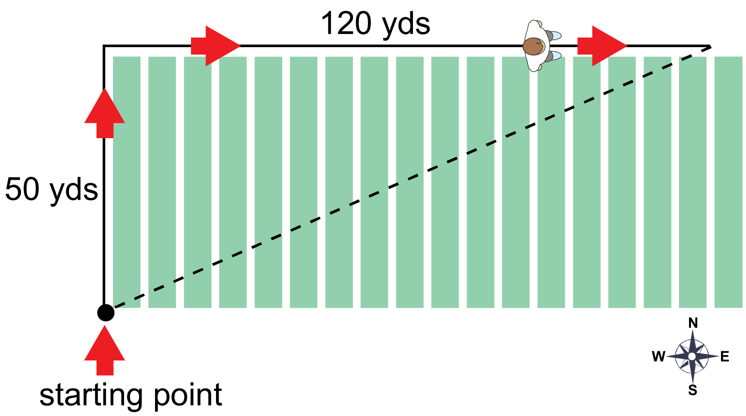


2. Wesley is pacing off a rectangular field. He walks 50 yards north and then 120 yards east. He suddenly gets thirsty and realizes that he has left his bottle of water back at the beginning point.

Hint: Remember Pythagoras.

How far is it from this point in a straight line to the starting point?

____



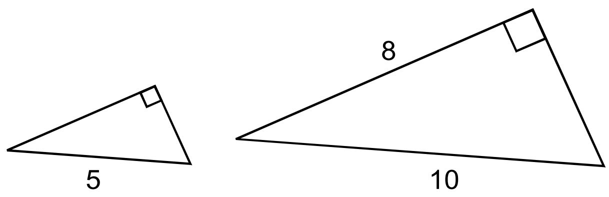
a. 70 yards
b. 170 yards
c. 130 yards
d. 125.4 yards

3. A small triangle and a large triangle are similar right triangles.

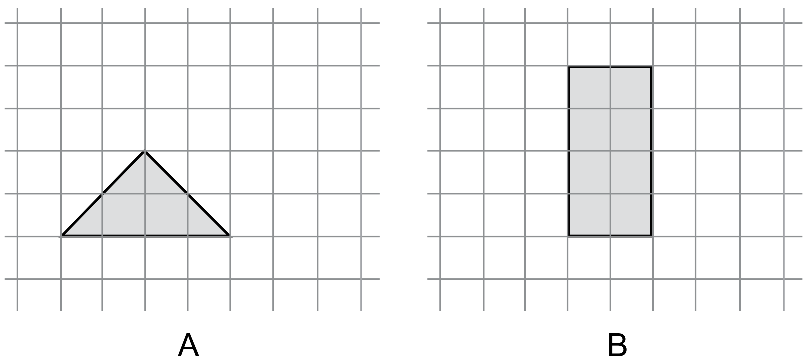
Hint: Remember that you need to know the height of the small triangle.

What is the area of the small triangle? ____________________________

a. 20 square feet
b. 6 square feet
c. 12 square feet
d. 60 square feet

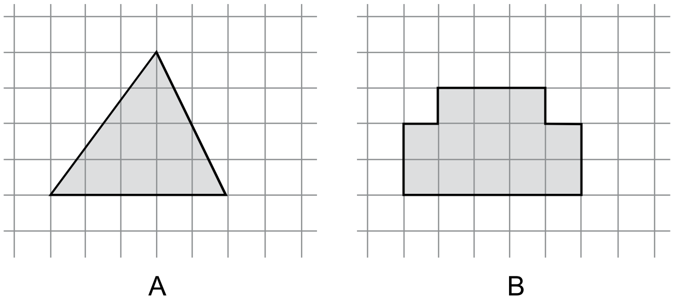


4. How would you compare the area of the triangle to the area of the rectangle?



a. They are equal in area.
b. The area of the triangle is $\frac{1}{2}$ the area of the rectangle.
c. The area of the triangle is twice the area of the rectangle.
d. You can't compare the areas of different figures.

5. What is the ratio of the area of the triangle to the area of the irregular polygon?



a. 20:13
b. 10:13
c. 13:10
d. 7:13

Answer the following.

6. Consider the following two squares. Notice that the sides of the larger square are twice as long as the sides of the smaller square.

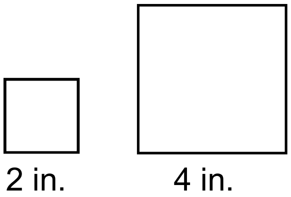
Find the ratio of the area of the large square to the area of the small

square. ____

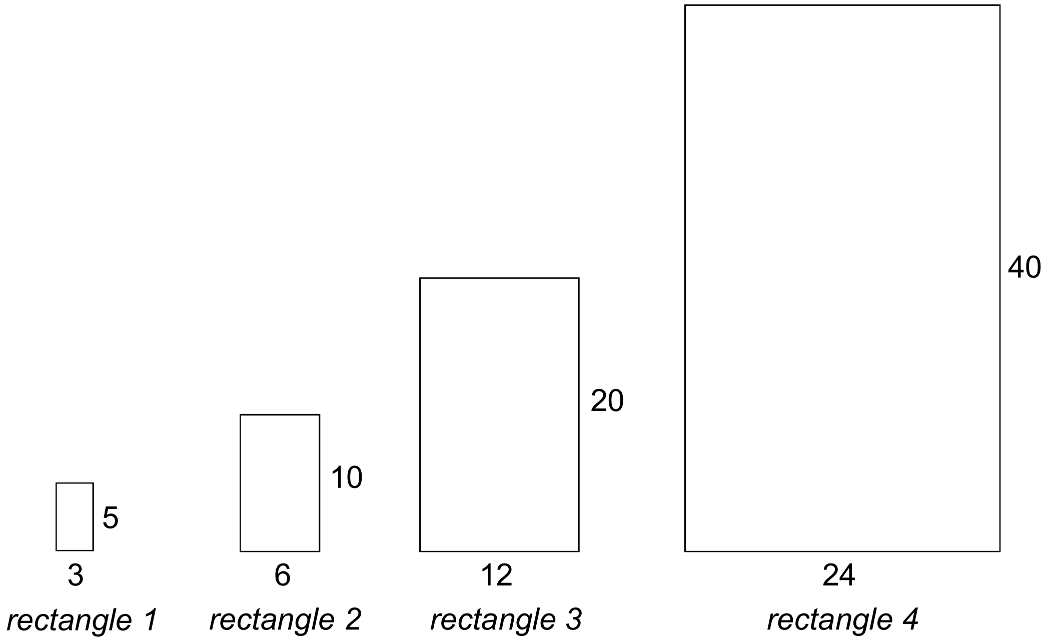
If we doubled the sides of the squares, do we double the areas?

____

By how much do the areas increase? ______________________



7. Consider these four rectangles, and note how the dimensions have been changed.



Find the following ratios:

$\frac{\text{area of rectangle 4}}{\text{area of rectangle 1}}$ $\frac{\text{area of rectangle 3}}{\text{area of rectangle 1}}$ $\frac{\text{area of rectangle 2}}{\text{area of rectangle 1}}$

________________ ________________ ________________

If we continue this pattern, what would be the ratio of the area of rectangle 5 to the area of rectangle 1? ____________________

8. Mr. Rodriguez is having some photos enlarged for his office. He wants to enlarge a photo that is 5 inches by 7 inches so the dimensions are *3 times larger* than the original.

How many times larger than the area of the original photo will the area of the

new photo be? ____

9. A circle has an area of 803.84 square meters.

Hint: To find the radius, you must work backward. You know that to find the area of a circle we use this formula:

Fill in what you know to figure out the radius.

$$
\begin{aligned}
A &= \pi r^2 \\
803.84 &\approx (3.14)r^2 \\
\frac{803.84}{3.14} &\approx \frac{3.14}{3.14}r^2 \\
256 &\approx r^2 \\
16 &\approx r
\end{aligned}
$$

divide both sides by 3.14

If you know the radius, you can now use the circumference formula that uses the radius ($C = 2\pi r$) to find the final answer.

What is the circumference of the circle?______________________

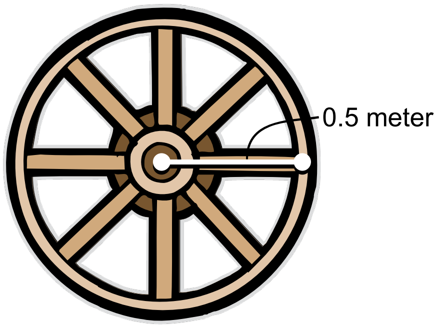
10. A sphere has a surface area of 1256 square meters. What is the radius of the sphere? Circle the letter of the correct answer.

a. 10 meters
b. 20 meters
c. 100 meters
d. 200 meters

11. On the wagon wheel below, it is 0.5 meter from the middle of the wheel to the outside of the wheel.

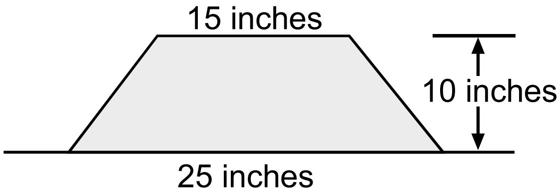
What is the circumference of the wheel? ____________________

How many meters will the wheel travel in 10 revolutions? ______



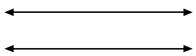
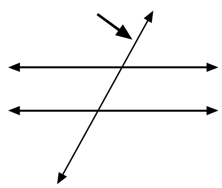
12. Pat is building a model railroad. He plans to build a stone bridge over an imaginary river. Each side of the bridge is shaped like a trapezoid. The dimensions of a side are given in the picture below. He needs to know the area so that he can buy enough rocks.

What is the total area of the 2 sides of the bridge? _____________



Practice

Match each definition with the correct term. Write each letter on the line provided.

______	1. to meet or cross at one point	A. congruent
______	2. two lines in the same plane that never meet 	B. intersect
______	3. figures or objects that are the same shape and the same size (≅)	C. line
______	4. a portion of a line that begins at a point and goes on forever in one direction (→)	D. line segment
______	5. a portion of a line that has a defined beginning and end (—)	E. parallel lines
______	6. a line that intersects two or more other (usually parallel) lines in the same plane 	F. ray
______	7. a straight line that is endless in length (↔)	G. transversal

Practice

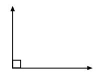
Match each graphic with the correct term. Write the letter on the line provided.

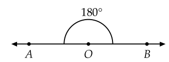
______ 1. acute angle

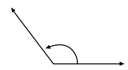
______ 2. obtuse angle

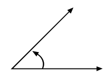
______ 3. right angle

______ 4. straight angle

A. 

B. 

C. 

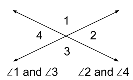
D. 

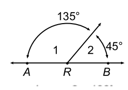
______ 5. complementary angles

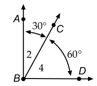
______ 6. measure of an angle

______ 7. supplementary angles

______ 8. vertical angles

A. 

B. 

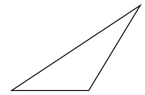
C. 

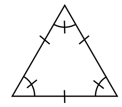
D. m∠

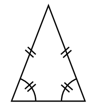
______ 9. equilateral triangle

______ 10. isosceles triangle

______ 11. obtuse triangle

A. 

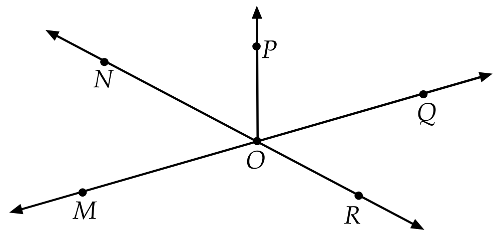
B. 

C. 

Unit Review

Part A

Use the figure below to find the following.



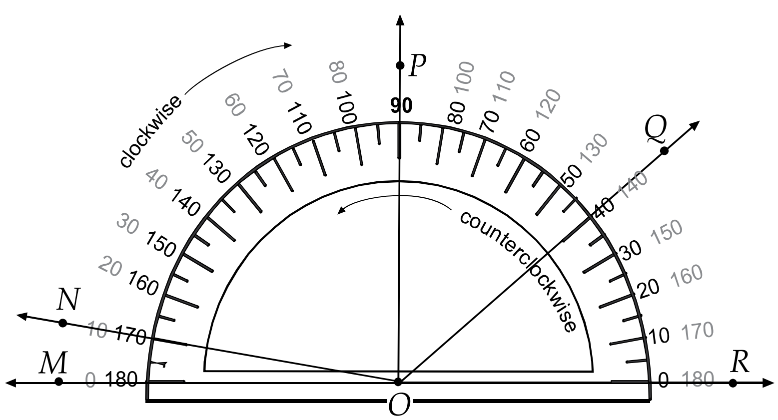
1. Write another name for $\overleftrightarrow{MQ}$. ______________________________

2. Name two lines that intersect. ______________________________

3. Are $\overrightarrow{OR}$ and $\overrightarrow{RO}$ opposite rays? ______________________________

4. Are $\overline{OR}$ and $\overline{RO}$ congruent? ______________________________

Use the **protractor** *below to find the* **measure of each angle**. *Then write whether the angle is* **acute**, **right**, *or* **obtuse**.



5. $\angle RON$ ______________________________

6. $\angle MON$ ______________________________

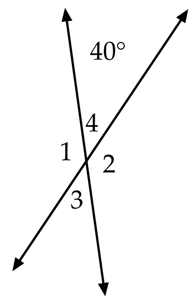
7. $\angle POQ$ ______________________________

Use the **angles on the protractor** *on the previous page to identify the following.*

8. a right angle ______________________________

9. a straight angle ______________________________

Use the figure below to find the following.



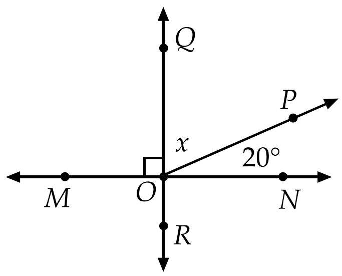
10. m∠1 ______________________________

11. m∠2 ______________________________

12. m∠3 ______________________________

13. Name two supplementary angles. ____________________

Use the figure below to find the following.

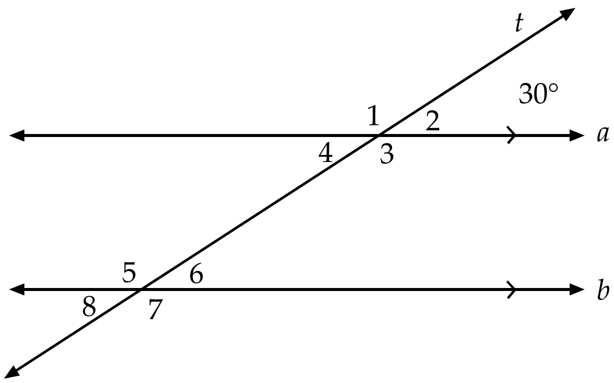


14. Find m∠x. ______________________________

15. Name a pair of complementary angles. ________________

16. Name a pair of vertical angles. ____________________

Use the figure below to find the following.



17. List each angle whose measure is 30 degrees. ____________________

18. Find m∠7. ____________________

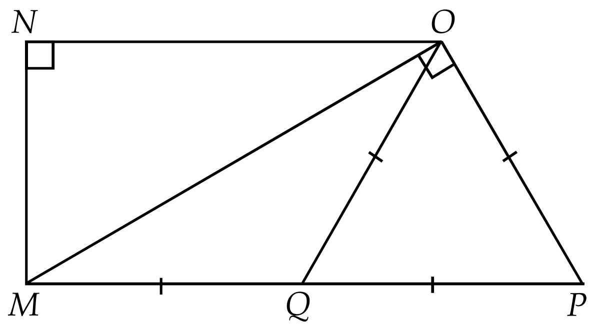
19. Name two parallel lines. ____________________

20. Name the transversal. ____________________

21. ∠4 and ∠6 are ____________________ interior angles.

22. ∠2 and ∠6 are ____________________ angles.

Use the figure below to find the following.



23. Name two right angles. ______________________________

24. Name an isosceles triangle. ______________________________

25. Name an equilateral triangle ______________________________

26. Name an obtuse triangle. ______________________________

27. Find the m $\angle NMO$ if the m $\angle NOM$ is 30 degrees (°). ____________

28. Find the length of *MP* if *MQ* is 10 inches. ____________________

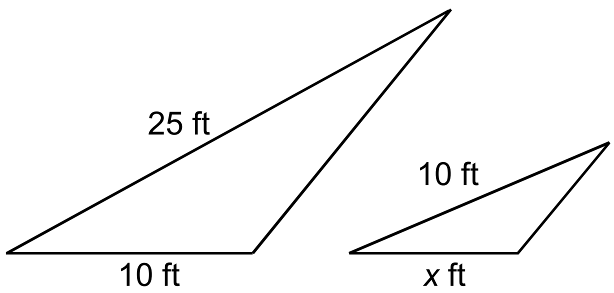
Unit Review

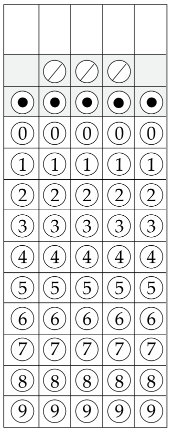
Part B

Numbers 29 and 30 are **gridded-response items**.
Write answers along the top of the grid and correctly mark them below.

The following shapes in numbers 29 and 30 are **similar**. *Set up a* **proportion** *to* **solve for** *x*.

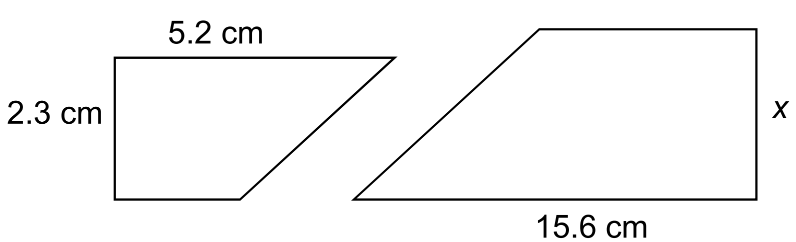
29.

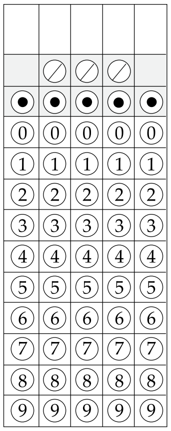




Mark your answers on the grid to the right.

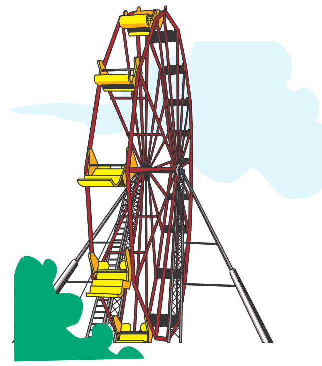
30.





Mark your answers on the grid to the right.

31. When a Ferris wheel casts a 20-meter shadow, a post 1.6 meters tall casts a 2.4-meter shadow.



First estimate based on what you know, then set up a proportion and solve.

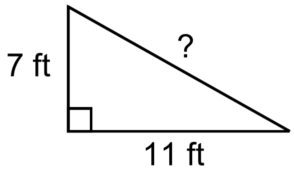
How tall is the Ferris wheel? ____________
Round to nearest tenth.

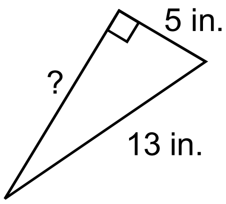
Is your answer reasonable? Explain. ______________________________

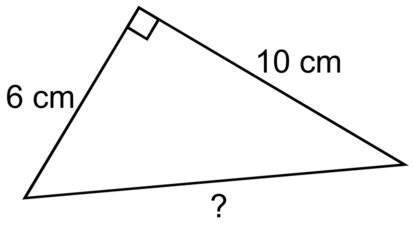
____

____

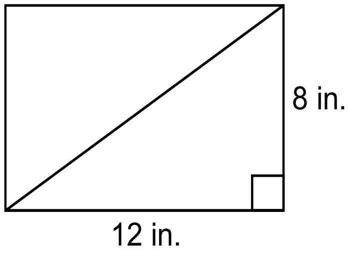
Using the **Pythagorean theorem** $a^2 + b^2 = c^2$ *to find the* **length of the missing side**. **Round to nearest hundredth.**

32. 

33. 

34. 

35. Find the diagonal of this rectangle. ______________________________

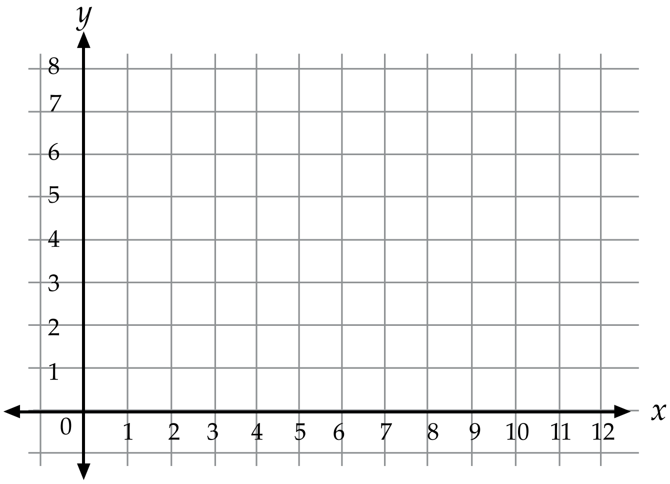


Complete the following.

36. Use the coordinate grid below to plot these points:
(1, 1), (9, 1), (9, 4), (1, 4)

Connect the points to form a rectangle.

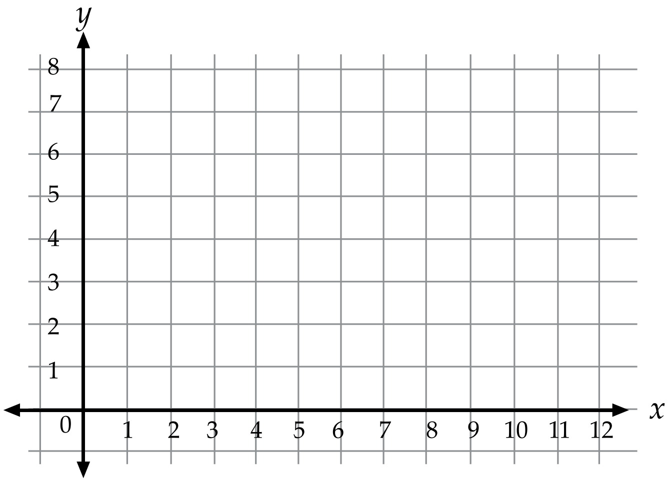
What is the area of the rectangle? ______________________________



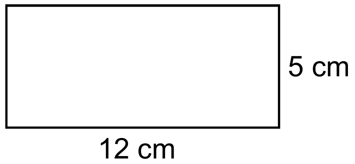
37. Use the coordinate grid below to plot these points.
(3, 1), (8, 1), (1, 4)

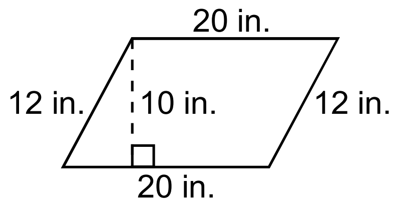
Connect the points to form a triangle.

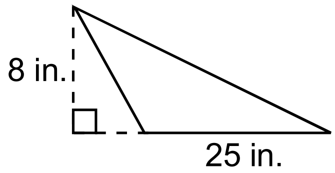
What is the area of the triangle? ______________________________

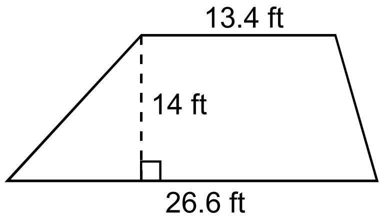


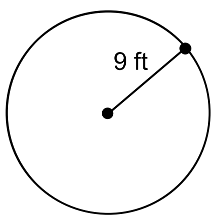
Use the **reference sheet** *in* **Appendix C** *to find the appropriate* **formulas**.
Then find the **area** *of the following figures.*

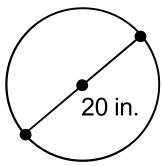
38. 

39. 

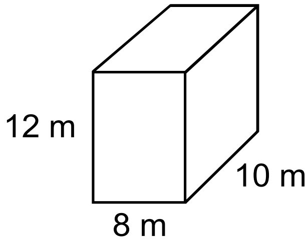
40. 

41. 

42. 

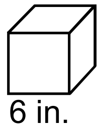
43. 

Use the **reference sheet** *in* **Appendix C** *to find the appropriate* **formulas**. *Then find the* **volume** *and* **surface area** *of the following figures.*



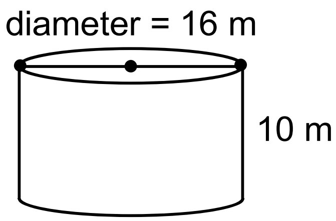
44. volume ____________________

45. surface area ____________________



46. volume ____________________

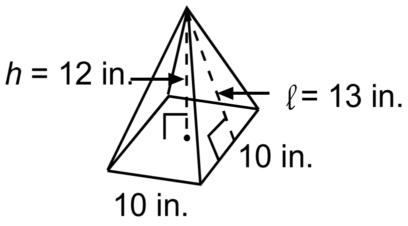
47. surface area ____________________



48. volume ____________________

49. surface area ____________________

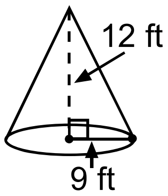
50.



volume ____________

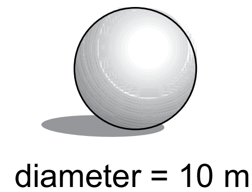
For the following two problems, leave your answer in π **form**.

51.



volume ____________

52.



volume ____________

Bonus Problems:

53. Refer to the picture on problem 50. Find the surface area if the slant height (ℓ) is 13 inches.

54. A circle has area 200.96 square inches. First find the radius and then find the circle's circumference.